Proceedings of the ASME 2009 Summer Bioengineering Conference (SBC2009)
June 17-21, Resort at Squaw Creek, Lake Tahoe, CA, USA

SBC2009- 206788

TOWARDS NON-INVASIVE ASSESSMENT OF THE ELASTIC PROPERTIES OF THE SPINAL AQUEDUCT

Bryn A. Martin[1], Thomas J. Royston[2], John N. Oshinski[3], Francis Loth[1]

[1]Department of Mechanical Engineering, University of Akron, Akron, OH

[2]Department of Mechanical and Industrial Engineering, University of Illinois at Chicago, Chicago, IL

[3]Department of Radiology and Biomedical Engineering, Emory University, Atlanta, GA

ABSTRACT
Analytical models are developed for cerebrospinal fluid (CSF) pressure wave propagation speed based on viscoelastic properties and geometry of the subarachnoid space (SAS). The models were compared to experimental tests on various compliant coaxial tube phantom models of the spinal SAS having different thicknesses and mechanical properties, with the ultimate goal of developing a noninvasive in vivo technique for determining the elastic properties of the spinal aqueduct. The in vitro models were constructed based on a healthy persons' spinal geometry and properties, and the generation of pressure waves in it mimics the in vivo mechanism. Results suggest that pressure wave propagation is a weighted combination of two types of wave motion inherent to the coupled fluid-structure system. Additionally, theoretical and experimental studies indicate that the spinal cord (SC) mechanical properties do not play a significant role in wave speed propagation through the system, whereas mechanical properties of the encasing structures of the spinal aqueduct (SA) do influence wave speed.

INTRODUCTION
Non-invasive assessment of material properties of biological tissues has played a role in assessment, diagnosis, and treatment of various pathologies of the vasculature and tissues. In particular, changes in viscoelastic properties of tissue may preclude or accompany disease, and in the case of biological systems conveying fluid, such property changes can be manifest in influencing the speed at which a pressure wave travels through the fluid conduit. For example, when intimal thickening occurs in the vascular system, the vessels are stiffened, thereby causing the wave speed in the diseased vessel to increase.

Similar to the vasculature, elastic properties of the SA may preclude or accompany pathology. CSF system compliance has been the subject of research in its relation to intracranial hypertension, Chiari malformation, syringomyelia (SM), and other craniospinal disorders. In the case of SM, it has been documented that significant tissue edema occurs before cyst formation [1]. The CSF velocity wave speed in the SA has been measured non-invasively with MR [2]. Overall, development of wave speed theory in the SA provides an understanding of how a clinically measured wave speed relates to SA elastic properties and pathology.

THEORY
Consider an infinite length isotropic cylindrical tube, representative of the spinal canal dura and encasing structures, having a radius "a", thickness "h", Young's modulus E, Poisson's ratio υ and density ρ. Along the tube central axis is a rod of radius a_c, representative of an assumed rigid SC. A compressible fluid occupies the space between the tube and the rod (spinal SAS). The tube is modeled using the Flugge-Byrne-Lur'ye (FBL) equations, which are valid for moderately thick cylindrical shells [3]. This theory assumes that resulting tube dynamic displacements are small, transverse normal stress acting on planes parallel to the shell middle surface are negligible, and fibers of the shell normal to the middle surface remain so after deformation and are themselves not subject to elongation. Consider axisymmetric mechanical wave propagation along this coupled system at frequency ω. Tube displacements "u" and "w" in the axial (x) and radial (r) directions, respectively, can be expressed in terms of inverse Fourier transforms in the axial wavenumber k_{0s} [4]:

$$u = \frac{1}{\sqrt{2\pi}} \sum_{s=0}^{\infty} \int_{-\infty}^{\infty} \overline{U}_{0s} e^{j(-k_{0s}x+\omega t+\pi/2)} \mathrm{d}k_{0s} , \tag{1a}$$

$$w = \frac{1}{\sqrt{2\pi}} \sum_{s=0}^{\infty} \int_{-\infty}^{\infty} \overline{W}_{0s} e^{-j(k_{0s}x-\omega t)} \mathrm{d}k_{0s} . \tag{1b}$$

One obtains the following by inserting these expressions into the homogeneous FBL equations and utilizing orthogonality of the axisymmetric s components:

$$\mathbf{L}\begin{bmatrix}\overline{U}_{0s}\\ \overline{W}_{0s}\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix},\ \text{with}\quad \mathbf{L}=\begin{bmatrix}L_{11} & L_{13}\\ L_{31} & L_{33}\end{bmatrix},\qquad L_{11}=-\Omega^2+(k_{0s}a)^2,$$

$$L_{13}=L_{31}=\upsilon(k_{0s}a)-\beta^2(k_{0s}a)^3,\quad \Omega=\omega a\sqrt{\rho(1-\upsilon^2)/E},$$

$$\text{and}\quad L_{33}=1+\beta^2\left(1+(k_{0s}a)^4\right)-\Omega^2-FL_i. \qquad (2)$$

Here, FL_i accounts for internal fluid loading from a compressible fluid in the coupled problem [4]. And, FL_i is dependent upon the rigid cord of radius $a_c < a$ that is assumed along the central axis, in that it affects the inner boundary condition for the Bessel functions that comprise the FL_i term. Also, $\beta^2 = h^2/12a^2$ and ω is the nondimensionalized frequency. Phase velocities $c_\#$ for the axisymmetric wave types in this system are found by setting the determinant of **L** in the above equation to zero to solve for propagating values of k_{0s}, with $c_\# = \omega/k_{0s}$. Due to the compressible fluid coupling, there are an infinite number of such solutions. However, only the two lowest phase speeds are considered here. If we assume fluid incompressibility and a thin structure encasing the SA (dura), these two wave speeds reduce to:

$$c_1=\sqrt{\frac{Eh}{2a\rho}}\sqrt{\left(1-\frac{a_c^2}{(a-h/2)^2}\right)},\quad c_2=\sqrt{\frac{E}{\rho(1-\upsilon^2)}}\ . \qquad (3)$$

Here, for c_1 in the limit as a_c goes to zero (no inner rigid SC) and h is very small, we recover the Moens-Korteweg expression (the term in the first square root). And, c_2 is in fact the expression for longitudinal (extensional in plane) wave motion in a thin walled shell.

An alternative expression to be solved for the phase speeds, under the same axisymmetric assumptions and allowing for a thick-walled dura, but assuming an *incompressible* fluid, can be found in [5]. Specifically, equation (15) of the reference can be solved for two axisymmetric phase speeds comparable to the above. In the discussion, predictions of these different models are compared.

METHODS

Eight flexible SA models were constructed in order to experimentally examine the presented equations. These models all had a 10 mm diameter SC and 16 mm inner diameter spinal column. The spinal column and cord were constructed with using a flexible silicone elastomer mixed at different base-to-hardener ratios to obtain different elastic properties for each model [6]. The Young's moduli of each SC and column were determined by performing stress-strain experiments on cylindrical samples of the material used for each model. A computer controlled pulsatile pump delivered a 5 ms flow pulse to each model while pressure was monitored with pressure catheters in the SAS separated by 36 cm. Wave speed was determined by dividing the sensor distance by the time delay between the arrival of the foot of the pressure wave at each sensor (estimated visually).

RESULTS

A summary of the in vitro test results is shown in Table I (h is the model thickness, E_s is the Young's modulus of the spinal cord, and E_d is the Young's modulus of the dura or spinal column). The experimentally measured wave speed, c_{exp}, was consistently within the bounds of the theoretically computed wave speed, c_1 and c_2 (Table I). Wave speed was little influenced by SC tension (data not given here), and had little change when the SC in M1-M4 was removed (Table I). Additional experiments conducted on M1, having SCs with different elasticity (E_s=0.02 to 2.2 MPa), did not substantially influence wave speed (avg. 39.3 m/s, S.D. 6.3 m/s). Overall, wave speed was found to be most influenced by the thickness and compliance of the dura.

DISCUSSION

It was apparent that the in vitro experimental technique for estimating wave speed was likely detecting both c_1 and c_2 type waves. Thus, the experimentally estimated phase speed is a weighted average of these two wave types. The presence of the cord did not alter the measured phase speed sufficiently to overcome uncertainties about the relative contributions of c_1 and c_2 to the measurement. The theories suggest that, even if the cord were rigid, which it was not (its Bulk modulus is close to water), it still has a small impact on predicted phase speeds, In comparing the theory presented in this paper, with other theories that ignore fluid compressibility and assume a thin-walled dura, we see that both of these assumptions do affect predicted results (Table I). Further study is needed to reconcile the effect of these assumptions and the differences in these model predictions. For example, more careful consideration of the effect of material viscosity on wave speed predictions for each theory is needed; this can be modeled via complex valued Young's moduli.

Table I. Summary of model parameters and results.

Model	M1	M2	M3	M4	M1[b]	M2[b]	M3[b]	M4[b]
h (mm)	17.9	12.0	5.2	2.3	17.9	12.0	5.2	2.3
E_s (MPa)	0.45	0.45	0.45	0.45	-----	no cord	-----	
E_d (MPa)	2.0	2.0	0.63	0.63	2.0	2.0	0.63	0.63
c_{exp} (m/s)[a]	40, 51	28,30	24, 24	14,16	36	45	26	18
c_1 (m/s) (eq. 2)	30	27	11	7.4	31	28	12	8.8
c_2 (m/s) (eq. 2)	54	53	29	28	55	54	29	29
c_1 (m/s) ref [5]	22	20	9.0	6.6	24	22	11	8.1
c_2 (m/s) ref [5]	37	39	24	26	45	45	26	27
c_1 (m/s) (eq. 3)	25	22	9.5	6.8	32	29	12	8.7
c_2 (m/s) (eq. 3)	50	50	28	28	50	50	28	28

[a] two experimental tests for cases M1 – M4, [b] without cord present

CONCLUSIONS

The presented work provides better understanding of the viscoelastic fluid-filled coaxial tube system forming the SA. The results indicate that the experimentally measured wave speed values relate to the predicted wave speeds for all of the systems, though there is some uncertainty in terms of the relative contributions of the different theoretically-predicted wave types in the experimental measurement. Further study is needed before such wave speed measurements can be used to accurately estimate SA mechanical properties, such as compliance.

REFERENCES

[1] Akiyama, Y., Koyanagi, I., Yoshifuji, K., Murakami, T., Baba, T., Minamida, Y., Nonaka, T., and Houkin, K., 2008, "Interstitial Spinal-Cord Oedema in Syringomyelia Associated with Chiari Type 1 Malformations," J Neurol Neurosurg Psychiatry, 79(10), pp. 1153-8.

[2] Kalata, W., Martin, B. A., Oshinski, J. N., Jerosch-Herold, M., Royston, T. J., and Loth, F., 2009, "Mr Measurement of Cerebrospinal Fluid Velocity Wave Speed in the Spinal Canal," IEEE Transactions on Biomedical Engineering, pp.

[3] Leissa, A. W., 1993, *Vibration of Shells*, Acous. Soc. Amer.,

[4] Yazicioglu, Y., Royston, T. J., Spohnholtz, T., Martin, B., Loth, F., and Bassiouny, H. S., 2005, "Acoustic Radiation from a Fluid-Filled, Subsurface Vascular Tube with Internal Turbulent Flow Due to a Constriction," J Acoust Soc Am, 118(2), pp. 1193-209.

[5] Cirovic, S., 2009, "A Coaxial Tube Model of the Cerebrospinal Fluid Pulse Propagation in the Spinal Column," J Biomech Eng, 131(2), pp. 021008.

[6] Martin, B. A., Kalata, W., Loth, F., Royston, T. J., and Oshinski, J. N., 2005, "Syringomyelia Hydrodynamics: An in Vitro Study Based on in Vivo Measurements," J Biomech Eng, 127(7), pp. 1110-20.